STA 624–MATLAB Exercise

Ruriko Yoshida

You are implementing an algorithm to solve the system of linear equations

$$Ax = b, \tag{1}$$

where $A \in \mathbb{R}^{n \times n}$ and x, $b \in \mathbb{R}^n$ using the LU decomposition. Remember the LU decomposition is the decoposition of matrix A into a lower and a upper triangular matrices, L, U such that

$$A = LU.$$

To solve the system in (1), one notices that:

$$Ax = (LU)x = L(Ux) = Ly = b,$$

where

$$y = Ux.$$

Thus to solve the system in (1), one has to solve the following two systems:

$$Ly = b, \tag{2}$$

$$Ux = y. \tag{3}$$

Why we use the LU decomposition to solve the system? This is because if we have a system with a lower or an upper triangular matrix, then it is very easy to solve via back- or forward-substitution. Here is an algorithm to solve the system in (1) via the LU decomposition.

Algorithm 1 (Solve the system in (1) via the LU decomposition)**.**

- **Input**: A, b.
- **Output**: x.
- **algorithm**:
  1. *Compute a lower and an upper triangular matrices* L, U, *such that* $A = LU$.
  2. *Set* $y_1 = b_1/L_{1,1}$.
  3. *For* $i = 2$ *to* n *do*
     (a) $sum = 0$.
     (b) *for* $j = 1$ *to* $i - 1$ *do*
         $sum = sum + L_{i,j} * y_j$
     (c) $y_i = (b_i - sum)/L_{i,i}$.
  4. *Set* $x_n = y_n/U_{n,n}$.
  5. *For* $i = 1$ *to* $n - 1$ *do*
     (a) $sum = 0$.
     (b) *for* $j = 0$ *to* $i - 1$ *do*

$sum = sum + U_{n-i,n-j} * x_{n-j}$

(c) $x_{n-i} = (y_{n-i} - sum)/U_{n-i,n-i}$

6. Return x*.*

Problem 2. Write a maple code in a file “LUsolve.m” to solve the system in (1) using Algorithm 1

Note. Please use “lualt.m” file to call a function “lualt” instead of using a function “lu” (there seems to be a bug in this function).