HOMEWORK 8
STA 624.01, Applied Stochastic Processes
Spring Semester, 2008

Due: Monday, March 21st, 2008

Readings: Section 3.3 of text

Regular Problems

1   (Lawler 3.3) Suppose X_t and Y_t are two independent Poisson processes with rate parameters λ_1 and λ_2, respectively, measuring of calls arriving at two different phones. Let $Z_t = X_t + Y_t$.
   (a) Show that Z_t is a poisson process. What is the rate parameter for Z?
   (b) What is the probability that the first call comes on the first phone?
   (c) Let T be the first time that at least one call has come from each of the two phones. Find the density and distribution function of the random variable T.

2   Measurements in a telephone exchange show that, during business hours, a telephone call in progress ends with constant rate $\lambda = 0.5$ per minute.
   (a) What is the distribution of the duration of a call?
   (b) What is the probability that a call lasts no longer than one minute?
   (c) What is the probability that a call that has lasted for one minute already, will be finished during the next minute?

3   A bridge is built to tolerate winds less than 75 miles per hour. If maximum wind speeds in storms are distributed like an exponential random variable with mean 40 miles per hour and storms arrive as a Poisson process with rate $\lambda = 2$ per year, what is the expected time until bridge failure?

4   Suppose $T_1, \cdots, T_n$ are independent randam variables each exponential with rates $b_1, \cdots, b_n$, respectively. Let $T = \min\{T_1, \cdots, T_n\}$. Show (in details) that

$$P(T_1 = T) = \frac{b_1}{b_1 + \cdots + b_n}.$$

5   A substitution model describes the process from which a sequence of characters of a fixed size from some alphabet changes into another set of traits. The Jukes-Cantor model is one of them. Go to `http://en.wikipedia.org/wiki/Models_of_DNA_evolution` and
   (1) Describe the model, i.e., what is the state space? Is it a CTMC? If so what is the rate matrix? In Biology, what does the model mean?
   (2) What is the limiting distribution? Please write in etails.
   (3) What is a distance between two sequences? Derive the distance under this model.