

**HOMEWORK 6**  
STA5724.01, Probability  
Fall Semester, 2008

**Due:** Friday, October 31st, 2008

**1** Suppose that the random variables  $X, Y, Z$  have the following joint pdf

$$f(x, y, z) = \begin{cases} 6 & \text{for } 0 < x < y < z < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the marginal univariate pdf's of  $X, Y$  and  $Z$ .

**2** Suppose that the random variables  $X, Y, Z$  have the following joint pdf

$$f(x, y, z) = \begin{cases} 2 & \text{for } 0 < x < y < 1 \text{ and } 0 < z < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Evaluate  $P(3X > Y | 1 < 4Z < 2)$ .

**3** Suppose  $X_1, X_2, X_3$  form a random sample of the three observation from a distribution having the following pdf:

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the pdf of the range of the sample.

**4** Julia collects figures from corn flakes packets. Each packet contains one figure, and  $n$  distinct figures make a complete set. Show that the expected number of packets that Julia needs to buy to collect a complete set is

$$n \sum_{i=1}^n 1/i.$$

**5** Suppose that  $X_1, X_2, \dots, X_n$  are independent identically distributed positive random variables with  $\mathbb{E}(X_i) = \mu < \infty$  and  $\mathbb{E}(X_i^{-1}) < \infty$ . Let  $S_n = \sum_{i=1}^n X_i$ . Show that for  $m \leq n$

$$\mathbb{E}(S_m/S_n) = m/n,$$

and

$$\mathbb{E}(S_m/S_n) = 1 + (m - n)\mu\mathbb{E}(S_n^{-1}).$$