HOMEWORK 7

STA5724.01, Probability
Fall Semester, 2007

Due: Friday, October 19th, 2007

1 Suppose n random variables $X_1, \cdots, X_n$ form a random sample from a discrete dustribution for which the p.f. is f. Determine the value of $Pr(X_1 = X_2 = \cdots = X_n)$.

2 Let X be a r.v. with a continuous distribution. Let $X_1 = X_2 = X$.
(a) Prove that both X_1 and X_2 have a continuous distribution.
(b) Prove that $\mathbf{X} = (X_1, X_2)$ does not have a continuous distribution.

3 Suppose $\mathbf{X}_1, \cdots \mathbf{X}_n$ are independent. Let $k < n$ and let $i_1, \cdots, i_k$ be distinct integers between 1 and n. Prove that $\mathbf{X}_{i_1}, \cdots \mathbf{X}_{i_k}$ are independent.

4 Let $\mathbf{X}$ be a random vector that is split into three parts $\mathbf{X} = (\mathbf{Y}, \mathbf{Z}, \mathbf{W})$. Suppose $\mathbf{X}$ has a continuous joint distribution with p.d.f. $f(\mathbf{y}, \mathbf{z}, \mathbf{w})$. Let $g_1(\mathbf{y}, \mathbf{z}|\mathbf{w})$ be the conditional p.d.f. of $(\mathbf{Y}, \mathbf{Z})$ given $\mathbf{W} = \mathbf{w}$ and let $g_2(\mathbf{y}|\mathbf{w})$ be the conditional p.d.f. of $\mathbf{Y}$ given $\mathbf{W} = \mathbf{w}$
Prove

$$g_2(\mathbf{y}|\mathbf{w}) = \int g_1(\mathbf{y}, \mathbf{z}|\mathbf{w}) dz.$$

5 Suppose n random variables $X_1, \cdots, X_n$ form a random sample from a continuous dustribution for which the p.d.f. is f. Determine the probability that at least k of these n r.v. will lie in a specified interval $a \leq x \leq b$.