Name________SOLUTION______________________

STA 524 Midterm 1

Probability

October 1st, 2007

There are five questions on this test. DO use calculators if you need them. “Do that one thing, and then that other thing” is not a valid answer. Show **ALL** your work (steps) to get an answer (on the exam or attach scratch papers). If you show only a number without steps, you will get no credit. There will be no bathroom break allowed. Divination to obtain answers is not allowed.

Obligatory Hitchhikers Guide to the Galaxy Notice: Don’t Panic!!!

You have 50 minutes to complete this test. Please ask me questions if a question needs clarification.

Each question is worth the same number of points.

“Monte Carlo has the right idea; fix a game where you are going to get people’s money, but the people don’t mind you getting it. A fellow can always get over losing money in a game of chance, but he seems so constituted that he can never get over money thrown away to a government in taxes.”

Will Rogers

“A Visit to Monte Carlo”

Question 1: Logic words

Let $A, B, C \subset \Omega$ be three events. Express in symbols the events:
We did in the class. See the lecture notes.

Question 2: Proof

We did in the class. See the lecture notes.

Question 3: Combinatorics

(a) How many solutions does the equation

$$x_1 + x_2 + x_3 = 11$$

have, where x_1, x_2, x_3 are nonnegative integers?

Balls and boxes trick. There are 3 boxes and 11 balls. So we have

$$\binom{11+3-1}{3-1}.$$

(b) How many different strings can be made by reordering the letters of the word **SUCCESS**?

By the Multinomial theorem, we have:

$$\binom{7}{3,2,1,1}.$$

Question 4: Independent events

There are two fair dice, one red and the other blue. Roll the two dice. Define the following events:

$$\begin{aligned} A &= \text{event that the red die is less than 4} \\ B &= \text{event that the red die is } 3, 4, \text{ or } 5 \\ C &= \text{event that the sum of the dice is } 5. \end{aligned}$$

(a) Show that $Pr(ABC) = Pr(A)Pr(B)Pr(C)$.

$$P(A) = 1/2,$$

$$P(B) = 1/2$$

$$P(C) = 4/36.$$

$$P(ABC) = 1/36.$$

Thus, by multiplication, we can show the result.

(b) Explain why this is not enough to show that events A, B, and C are independent.

$$P(AB) = 1/6 \neq 1/4 = P(A)P(B).$$

(c) Determine whether or not these three events are independent.
By (b) it is not independent.

Question 5: Conditional Probability and Bayes' Theorem

Fred has just found out that there is a 1/3 chance that he has contracted a viral infection the last time he went to the Twilight Zone. The only effect of the disease, if he has contracted, is that if he has any children, they each have a 1/4 chance of having some disease.

(a) Fred married Martha the next year. What is the probability that their first two children will have not the disease? Assume that among the uninfected population, the probability of having the disease is zero.

Define S_1 be the event that the first child has the disease, S_2 be the event that the 2nd child has disease and F be the event that Fred has infected in the zone. Then we have:

$$\begin{aligned} P(S_1^c S_2^c) &= P(S_1^c S_2^c|F)P(F) + P(S_1^c S_2^c|F^c)P(F^c) \\ &= P(S_1^c|F)P(S_2^c|F)P(F) + P(S_1^c|F^c)P(S_2^c|F^c)P(F^c) \\ &= 41/48. \end{aligned}$$

(b) Let N be the event that the first and 2nd children does not have disease. Then

$$P(N|F) = P(S_1^c|F)P(S_2^c|F) = 3/4 \cdot 3/4 = 9/16.$$

(c) By Bayes' theorem, we have

$$P(F|N) = \frac{P(N|F)P(F)}{P(N|F)P(F) + P(N|F^c)P(F^c)} = \frac{P(N|F)P(F)}{P(N)} = 41/256.$$