STA 291
Spring 2009

LECTURE 20

TUESDAY, April 21

Administrative Notes

- **11/12 Hypothesis Testing**
  - 11.2 Test for the Population Mean
  - **12.3 Inference about a Population Proportion**
  - **12.1 Small Sample Inference about a Population Mean**

- *This week, a make-up online homework is posted: it can be used to substitute the lowest homework grade (or, a missing homework grade). It consists of review questions from Chapters 6-11*

Review: Test for the Population Mean

	One-Sided Tests		Two-Sided Test
Null Hypothesis	$H_0 : \mu = \mu_0$		
Research Hypothesis	$H_1 : \mu < \mu_0$	$H_1 : \mu > \mu_0$	$H_1 : \mu \neq \mu_0$
Test Statistic	$z = \dfrac{\bar{X} - \mu_0}{s / \sqrt{n}}$		
p-value	$P(Z < z_{obs})$	$P(Z > z_{obs})$	$2 \cdot P(Z > \lvert z_{obs} \rvert)$

12.3 Large Sample Significance Test for a Population Proportion

	One-Sided Tests		Two-Sided Test
Null Hypothesis	$H_0 : p = p_0$		
Research Hypothesis	$H_1 : p < p_0$	$H_1 : p > p_0$	$H_1 : p \neq p_0$
Test Statistic	$z_{obs} = \dfrac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}}$		
p-value	$P(Z < z_{obs})$	$P(Z > z_{obs})$	$2 \cdot P(Z > \lvert z_{obs} \rvert)$

Significance Test for a Proportion

Assumptions

- What type of data?
  - *Qualitative*
- Which sampling method has been used?
  - *Random sampling*
- What is the sample size?
  - $n \geq 20$ *if is* p_0 *between 0.25 and 0.75*
  - *In general (rule of thumb): Choose* n *such that* $n \geq 5/p_0$ *and* $n \geq 5/(1 - p_0)$

Significance Test for a Proportion

Hypotheses

- Null hypothesis H_0: $p = p_0$ where p_0 is *a priori* (beforehand) specified

- Alternative hypotheses can be one-sided or two-sided

- Again, two-sided is more common

Significance Test for a Proportion

$$z_{obs} = \frac{\text{value from the data} - \text{value from } H_0}{\text{standard error of the estimator used}}$$

$$= \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$$

P-Value

- Calculation is exactly the same as for the test for a mean
- Find one- or two-sided tail probabilities using Table B3

Example

- Let $\boldsymbol{p}$ denote the proportion of Kentuckians who think that government environmental regulations are too strict

- Test H_0: $\boldsymbol{p} = 0.5$ against a two-sided alternative using data from a telephone poll of 834 people in which 26.6% said regulations were too strict

  1. Calculate the test statistic
  2. Find the *p*-value and interpret
  3. Using alpha=0.01, can you determine whether a majority or minority think that environmental regulations are too strict, or is it plausible that $\boldsymbol{p} = 0.5$?
  4. Construct a 99% confidence interval. Explain the advantage of the confidence interval over the test.

12.1 Small Sample Confidence Interval for a Mean

- What if we want to make inference about the population mean, but our sample size is not big enough to meet the minimal sample size requirement n>25 to apply the Central Limit Theorem?
- Confidence intervals are constructed in the same way as before, but now we are using ***t-values*** instead of ***z-values***

12.1 Small Sample Confidence Interval for a Mean

- For a random sample ***from a normal distribution***, a 95% confidence interval for μ is

$$\bar{x} \pm t_{0.025} \frac{s}{\sqrt{n}}$$

- where $t_{0.025}$ is a t-score (instead of z-score) from Table B4 (p. B-9) or better, from a site like *surfstat:*
- http://www.anu.edu.au/nceph/surfstat/surfstat-home/tables/t.php
- degrees of freedom are $df = n - 1$

Attendance Survey Question #20

- ***On a 4”x6” index card***
  - Please write down your name and section number
  - Today’s Question: