STA 291 Spring 2009

LECTURE 19 THURSDAY, 14 April

Administrative Notes

- 11 Hypothesis Testing
 - 11.1 Concepts of Hypothesis Testing
 - 11.2 Test for the Population Mean
- Online homework due this Sat
- Suggested Reading

 Study Tools or Textbook Chapter 11.2
- *Suggested* problems from the textbook: 11.7, 11.8, 11.9, 11.13, 11.14, 11.15

Review: 11.1 Significance Tests

- A significance test is used to find evidence *against* a hypothesis
- The sampling distribution helps quantify the evidence ("*p*-value")
- Enough evidence against the hypothesis: Reject the hypothesis.
- Not enough evidence: No conclusion.

Elements of a Significance Test

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- Assumptions
 - Type of data, population distribution, sample size
- Hypotheses
 - Null and alternative hypothesis

- Test Statistic
 - Compares point estimate to parameter value under the null hypothesis
- P-value
 - Uses sampling distribution to quantify evidence against null hypothesis
 - Small P is more contradictory
- Conclusion
 - Report P-value
 - Make formal rejection decision (optional)

p-Value

- How unusual is the observed test statistic when the null hypothesis is assumed true?
- The *p-value* is the probability, assuming that H_0 is true, that the test statistic takes values at least as contradictory to H_0 as the value actually observed
- The smaller the *p*-value, the more strongly the data contradict H_0

Decisions and Types of Errors in Tests of Hypotheses

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• Terminology:

 $-\alpha$ -level (significance level) is a number such that one rejects the null hypothesis if the *p*-value is less than or equal to it.

– Often, α =0.05

– Choice of the α -level reflects how cautious the researcher wants to be ("acceptable risk")

– Significance level α needs to be chosen **before** analyzing the data

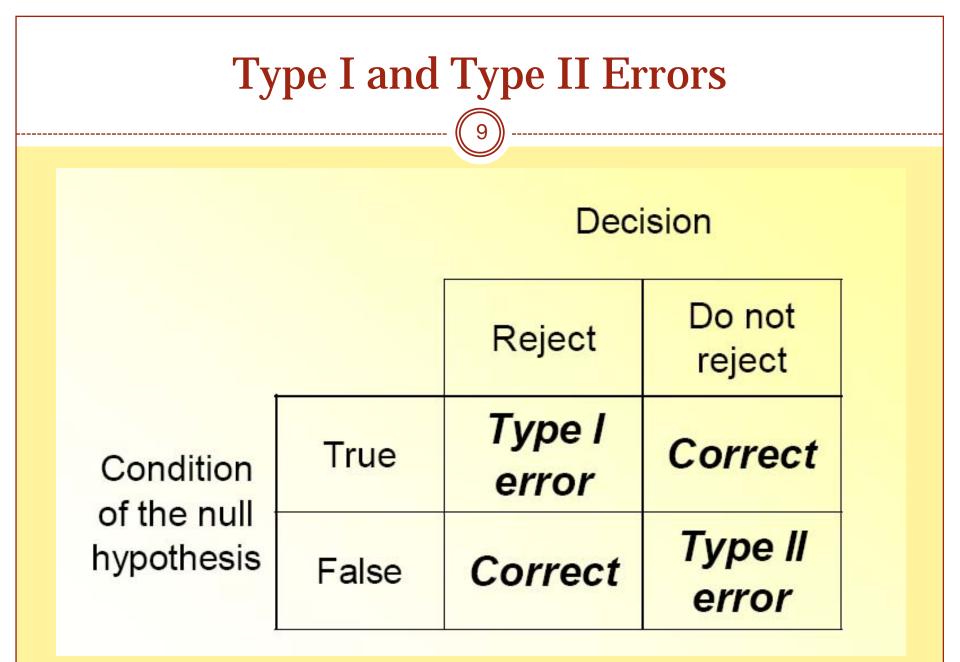
Decisions and Types of Errors in Tests of Hypotheses

• More Terminology:

– The rejection region is a range of values such that if the test statistic falls into that range, we decide to reject the null hypothesis in favor of the alternative hypothesis

Type I and Type II Errors

- Type I Error: The null hypothesis is rejected, even though it is true.
- Type II Error: The null hypothesis is not rejected, even though it is false.



Type I and Type II Errors

- Terminology:
 - $-\alpha$ = **Probability of a Type I error**
 - $-\beta = Probability of a Type II error$
 - Power = 1 Probability of a Type II error
- The smaller the probability of Type I error, the larger the probability of Type II error and the smaller the power
- If you ask for very strong evidence to reject the null hypothesis, it is more likely that you fail to detect a real difference

Type I and Type II Errors

- In practice, α is specified, and the probability of Type II error could be calculated, but the calculations are usually difficult
- How to choose α ?
- If the consequences of a Type I error are very serious, then α should be small.
- For example, you want to find evidence that someone is guilty of a crime
- In exploratory research, often a larger probability of Type I error is acceptable
- If the sample size increases, both error probabilities decrease

Example

- The mean score for all high school seniors taking a college entrance exam equals 500.
- A study is conducted to see whether a different mean applies to those students born in a foreign country.
- For a random sample of 100 of such students, the mean and standard deviation on this exam equal 508 and 100.
- Q: Is the mean significantly different from 500 for international students?

Assumptions

- What type of data?
 - Quantitative
- What is the population distribution?
 - No special assumptions. The test refers to the
- population mean of the quantitative variable.
- Which sampling method has been used?
 - Random sampling
- What is the sample size?

Minimum sample size of n=30 to use Central Limit
 Theorem with estimated standard deviation

Hypotheses

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- The null hypothesis has the form H_0 : $\mu = \mu_0$, where μ_0 is an *a priori* (before taking the sample) specified number like 0 or 5.3
- The most common alternative hypothesis is

H₁: $\mu \neq \mu_0$

• This is called a *two-sided* hypothesis, since it includes values falling above and below the null hypothesis

Test Statistic

- The hypothesis is about the population mean
- So, a natural test statistic would be the sample mean
- The sample mean has, for sample size of at least *n*=25, an approximately normal sampling distribution
- The parameters of the sampling distribution are, under the null hypothesis,
- Mean = μ_0 (that is, the sampling distribution is centered around the hypothesized mean)

– Standard error =
$$\frac{\sigma}{\sqrt{n}}$$
, estimated by $\frac{s}{\sqrt{n}}$

Test Statistic

- Then, the *z*-score has a standard normal distribution $\frac{x - \mu_0}{\sigma}$
- The *z*-score measures how many estimated standard errors the sample mean falls from the hypothesized population mean
- The farther the sample mean falls from μ_0 , the larger the absolute value of the *z* test statistic, and the stronger the evidence against the null hypothesis

p-Value

• The *p*-value has the advantage that different test results from different tests can be compared: The *p*-value is always a number between 0 and 1

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- The *p*-value can be obtained from Table B3: It is the probability that a standard normal distribution takes values more extreme than the observed *z* score
- The smaller the *p*-value is, the stronger the evidence against the null hypothesis and in favor of the alternative hypothesis
- Round *p*-value to two or three significant digits

Example

- The mean score for all high school seniors taking a college entrance exam equals 500. A study is conducted to see whether a different mean applies to those students born in a foreign country. For a random sample of 100 of such students, the mean and standard deviation on this exam equal 508 and 100.
- 1. Set up hypotheses for a significance test.
- 2. Compute the test statistic.
- 3. Report the *P*-value, and interpret.
- 4. Can you conclude that the population mean for students born in a foreign country equals 500? Why or why not?
- 5. Make a decision about H_0 , using α =0.05.

One-Sided Tests of Hypotheses

- Recall: The research hypothesis is usually the alternative hypothesis
- This is the hypothesis that we want to prove by rejecting the null hypothesis
- Assume that we want to prove that is larger than a particular number μ_0
- Then, we need a one-sided test with hypotheses:

H₀:
$$\mu = \mu_0$$
 vs. H₁: $\mu > \mu_0$

One-Sided Alternative Example

• Example: Usually, Americans eat 2.5 pounds of turkey on Thanksgiving day.

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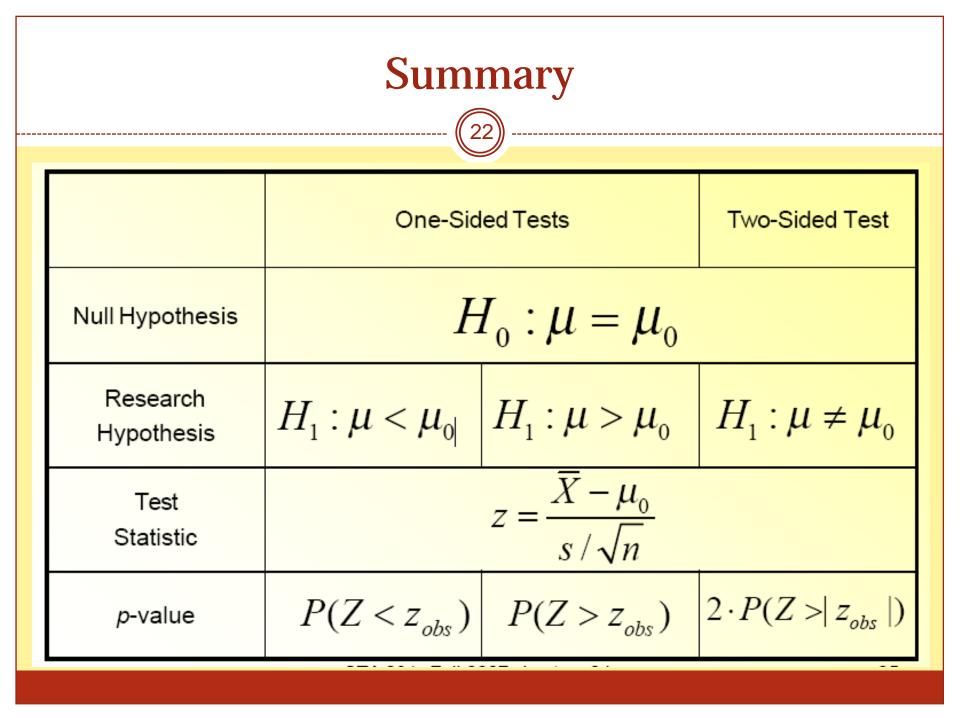
- You want to prove that this year, that figure is too high—that Americans are cutting back.
- You sample *n* = 40 Americans, asking how much they eat.
- Null hypothesis: H_0 : $\mu = 2.5$
- Alternative hypothesis: H_1 : $\mu < 2.5$

Two-Sided versus One-Sided

Two-sided tests are more common

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- Look for formulations like
 - "test whether the mean has *changed*"
 - "test whether the mean has increased"
 - "test whether the mean is the same"
 - "test whether the mean has decreased"



Attendance Survey Question #19

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• On a 4"x6" index card

– Please write down your name and section number
– Today's Question: