STA 291
Spring 2009

LECTURE 14
THURSDAY, 12 March

Binomial Distribution (review)

- The probability of observing k successes in n independent trials is

$$P(X = k) = \binom{n}{k} p^k q^{n-k}, \text{ for } k = 0,1,\cdots,n$$

Helpful resources (besides your calculator):

- Excel:

Enter	Gives
=BINOMDIST(4,10,0.2,FALSE)	0.08808
=BINOMDIST(4,10,0.2,TRUE)	0.967207

- Table 1, pp. B-1 to B-5 in the back of your book

Binomial Probabilities

We are choosing a random sample of n = 7 Lexington residents—our random variable, C = number of Centerpointe supporters in our sample. Suppose, p = P (Centerpointe support) ≈ 0.3. Find the following probabilities:

a) $P(C = 2)$

b) $P(C < 2)$

c) $P(C \leq 2)$

d) $P(C \geq 2)$

e) $P(1 \leq C \leq 4)$

What is the *expected* number of Centerpointe supporters, μ_C?

Center and Spread of a Binomial Distribution

- Unlike generic distributions, you don't need to go through using the ugly formulas to get the mean, variance, and standard deviation for a binomial random variable (although you'd get the same answer if you did):

$$\mu = np$$

$$\sigma^2 = npq$$

$$\sigma = \sqrt{npq}$$

Continuous Probability Distributions

- For continuous distributions, we can not list all possible values with probabilities
- Instead, probabilities are assigned to intervals of numbers
- The probability of an individual number is 0
- Again, the probabilities have to be between 0 and 1
- The probability of the interval containing all possible values equals 1
- Mathematically, a continuous probability distribution corresponds to a (density) function whose integral equals 1

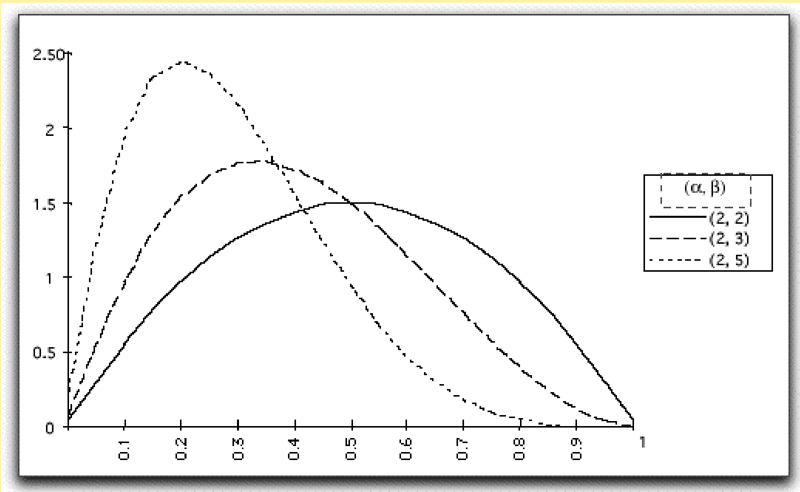
Continuous Probability Distributions: Example

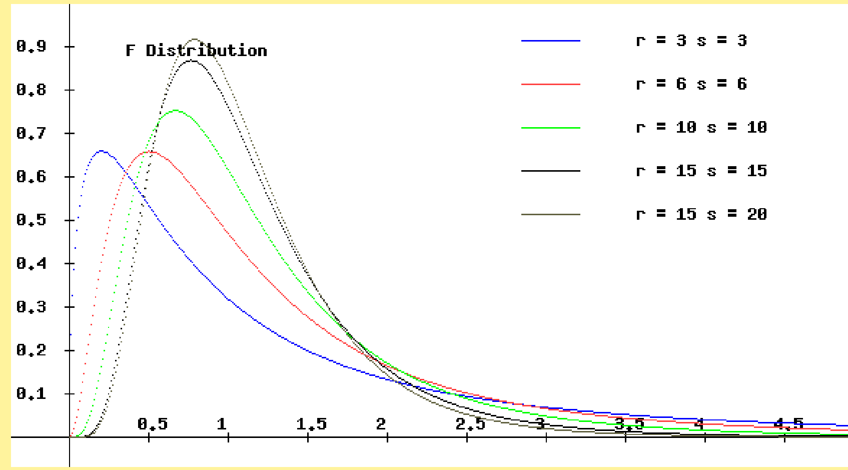
- Example: X=Weekly use of gasoline by adults in North America (in gallons)
- $P(6<X<9)=0.34$
- The probability that a randomly chosen adult in North America uses between 6 and 9 gallons of gas per week is 0.34
- Probability of finding someone who uses exactly 7 gallons of gas per week is 0 (zero)—might be *very close* to 7, but it won't be exactly 7.

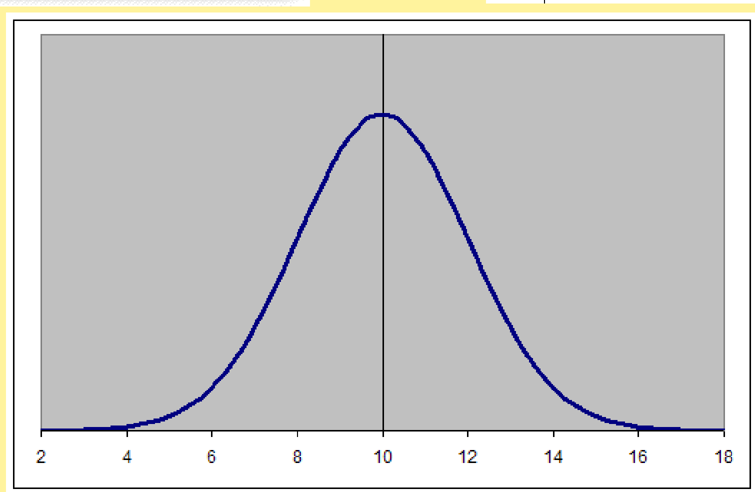
Graphs for Probability Distributions

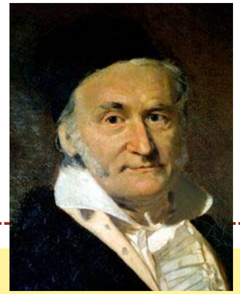
- Discrete Variables:
  - Histogram
  - Height of the bar represents the probability
- Continuous Variables:
  - Smooth, continuous curve
  - Area under the curve for an interval represents the probability of that interval

Some Continuous Distributions

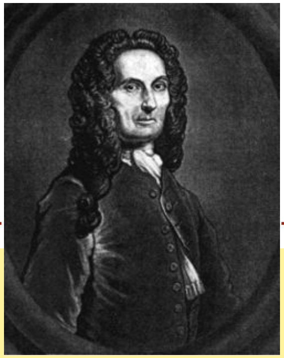








The Normal Distribution



- Carl Friedrich Gauß (1777-1855), ***Gaussian Distribution***
- Normal distribution is perfectly ***symmetric and bell-shaped***
- Characterized by two parameters: ***mean μ*** and ***standard deviation σ***
- The ***68%-95%-99.7% rule*** applies to the normal distribution; that is, the probability concentrated within 1 standard deviation of the mean is always 0.68; within 2, 0.95; within 3, 0.997.
- The ***IQR ≈ 4/3σ rule*** also applies

Normal Distribution Example

- Female Heights: women between the ages of 18 and 24 average 65 inches in height, with a standard deviation of 2.5 inches, and the distribution is approximately normal.
- Choose a woman of this age at random: the probability that her height is between $\mu-\sigma=62.5$ and $\mu+\sigma=67.5$ inches is ______%?
- Choose a woman of this age at random: the probability that her height is between $\mu-2\sigma=60$ and $\mu+2\sigma=70$ inches is ______%?
- Choose a woman of this age at random: the probability that her height is greater than $\mu+2\sigma=70$ inches is ______%?

Normal Distributions

- So far, we have looked at the probabilities within one, two, or three standard deviations from the mean

$$(\mu \pm \sigma, \mu \pm 2\sigma, \mu \pm 3\sigma)$$

- How much probability is concentrated within 1.43 standard deviations of the mean?
- More generally, how much probability is concentrated within *z* standard deviations of the mean?

Calculation of Normal Probabilities

Table 3 (page B-8) :

- Gives amount of probability between 0 and z, the *standard normal* random variable.

Example exercises:

- p. 253, #8.15, 21, 25, and 27.

So what about the "z standard deviations of the mean" stuff from last slide?

Attendance Question #14

Write your name and section number on your index card.

Today's question: