STA 291
Spring 2009

LECTURE 18
TUESDAY, 3 MARCH

Le Menu

- **6 Probability (Review, mostly)**
- **7 Random Variables and Discrete Probability Distributions**

Homework

- Suggested problems from the textbook:

  7.2,7.4 (“mark” = “score”), 7.14, 7.26, and 7.28

Conditional Probabilities—Another Perspective

Example: Smoking and Lung Disease I

Joint Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12	.19	.31
Nonsmoker	.03	.66	.69
Column Totals	.15	.85	1.00

Conditional Probabilities—Another Perspective

Example: Smoking and Lung Disease I

Joint Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12	.19	.31
Nonsmoker	.03	.66	.69
Column Totals	.15	.85	1.00

Example: Smoking and Lung Disease II

Conditional Row Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12/.31 =.39	.19/.31 =.61	.31/.31 =1.00
Nonsmoker	.03/.69 =.04	.66/.69 =.96	.69/.69 =1.00
Smokers and Nonsmokers	.15	.85	1.00

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probabilities—Another Perspective

Example: Smoking and Lung Disease I

Joint Probabilities	Lung Disease	Not Lung Disease	*Row Totals*
Smoker	.12	.19	.31
Nonsmoker	.03	.66	.69
Column Totals	.15	.85	1.00

Example: Smoking and Lung Disease III

Conditional Column Probabilities	Lung Disease	Not Lung Disease	*Lung Disease and Not Lung Disease*
Smoker	.12/.15 =.80	.19/.85 =.22	.31
Nonsmoker	.03/.15 =.20	.66/.85 =.78	.69
Column Totals	.15/.15 =1.00	.85/.85 =1.00	1.00

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Terminology

- $P(A \cap B) = P(A,B)$ joint probability of A and B (of the intersection of A and B)
- $P(A|B)$ conditional probability of A given B
- $P(A)$ marginal probability of A

Chapter 7: Random Variables

- A variable X is a **random variable** if the value that X assumes at the conclusion of an experiment cannot be predicted with certainty in advance.
- There are two types of random variables:
  - **Discrete**: the random variable can only assume a finite or countably infinite number of different values (almost always a count)
  - **Continuous**: the random variable can assume all the values in some interval (almost always a physical measure, like distance, time, area, volume, weight, …)

Examples

Which of the following random variables are discrete and which are continuous?

a. X = Number of houses sold by real estate developer per week?

b. X = Number of heads in ten tosses of a coin?

c. X = Weight of a child at birth?

d. X = Time required to run a marathon?

Properties of Discrete Probability Distributions

Definition: A Discrete probability distribution is just a list of the possible values of a r.v. X, say (x_i) and the probability associated with each $P(X=x_i)$.

Properties:

1. All probabilities non-negative.
2. Probabilities sum to _____ .

$$P(x_i) \geq 0$$

$$\sum P(x_i) = 1$$

Example

The table below gives the # of days of sick leave for 200 employees in a year.

Days	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Number of Employees	20	40	40	30	20	10	10	30

An employee is to be selected at random and let X = # days of sick leave.

a.) Construct and graph the probability distribution of X

b.) Find $P(X \leq 3)$

c.) Find $P(X > 3)$

d.) Find $P(3 \leq X \leq 6)$

Population Distribution vs. Probability Distribution

- If you select a subject randomly from the population, then the probability distribution for the value of the random variable *X* is the relative frequency (population, if you have it, but usually approximated by the sample version) of that value

Cumulative Distribution Function

Definition: The *cumulative distribution function*, or *CDF* is

$$F(x) = P(X \leq x).$$

Motivation: Some parts of the previous example would have been easier with this tool.

Properties:

1. For any value x, $0 \leq F(x) \leq 1$.
2. If $x_1 < x_2$, *then* $F(x_1) \leq F(x_2)$
3. $F(-\infty) = 0$ *and* $F(\infty) = 1$.

Example

Let X have the following probability distribution:

X	2	4	6	8	10
$P(x)$	.05	.20	.35	.30	.10

a.) Find $P\ (\ X \leq 6\ )$

b.) Graph the cumulative probability distribution of X

c.) Find $P\ (\ X > 6)$

Attendance Question #12

Write your name and section number on your index card.

Today's question: